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W.O.

If $\log_e \cos(\theta - i\phi) = A + iB$

D1(H) Maths
Paper-I, Group-A
Hyperbolic function
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prove $A = \frac{1}{2}\log_e \frac{1}{2}(\cos 2\theta + \cosh 2\phi)$

$\phi = \frac{1}{2}\log \frac{\sin(\theta + B)}{\sin(\theta - B)}$

$\because \log_e \cos(\theta - i\phi) = A + iB$

$\therefore \cos(\theta - i\phi) = e^{A+iB}$

or $\cos\theta \cos i\phi + \sin\theta \cdot \sin i\phi = e^A \cdot e^{iB}$

or $\cos\theta \cosh\phi + i\sin\theta \cdot \sinh\phi = e^A(\cos B + i\sin B)$

Equating real and imaginary parts

$\cos\theta \cosh\phi = e^A \cos B$ (1)

$\sin\theta \sinh\phi = e^A \sin B$ (2)

Squaring (1) and (2)

$\cos^2\theta \cosh^2\phi = e^{2A}\cos^2 B$ — (3)

$\sin^2\theta \sinh^2\phi = e^{2A}\sin^2 B$ — (4)

Adding, $\cos^2\theta \cosh^2\phi + \sin^2\theta \cdot \sinh^2\phi = e^{2A}(\cos^2 B + \sin^2 B)$

or, $2\cos^2\theta \, 2\cosh^2\phi + 2\sin^2\theta \, 2\sinh^2\phi = e^{2A} \times 4$

or, $(1+\cos 2\theta)(1+\cosh 2\phi) + (1-\cos 2\theta)(\cosh 2\phi - 1) = e^{2A} \times 4$

or, $\cancel{1} + \cosh 2\phi + \cos 2\theta + \cancel{\cos 2\theta \cosh 2\phi} + \cosh 2\phi$
$\cancel{-1} - \cancel{\cos 2\theta \cosh 2\phi} + \cos 2\theta = e^{2A} \times 4$

or, $2\cosh 2\phi + 2\cos 2\theta = e^{2A} \times 4$

or, $\cosh 2\phi + \cos 2\theta = 2e^{2A}$

$\therefore e^{2A} = \frac{1}{2}\{\cosh 2\phi + \cos 2\theta\}$

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or $\log e^{2A} = \log\left\{\frac{1}{2}(\cosh 2\phi + \cos 2\theta)\right\}$

$\therefore 2A \log e = \log\left\{\frac{1}{2}(\cosh 2\phi + \cos 2\theta)\right\}$

$2A \cdot 1 = \log\left\{\frac{1}{2}(\cosh 2\phi + \cos 2\theta)\right\}$

$\therefore A = \frac{1}{2}\log\left\{\frac{1}{2}(\cosh 2\phi + \cos 2\theta)\right\}$ proved 1st part

2nd part: $\rightarrow$

(2) $\div$ (1) gives

$$\frac{e^A \sin B}{e^A \cos B} = \frac{\sin\theta \sinh\phi}{\cos\theta \cosh\phi}$$

or $\dfrac{\sin B}{\cos B} = \dfrac{\sin\theta \sinh\phi}{\cos\theta \cosh\phi}$

or $\dfrac{\sinh\phi}{\cosh\phi} = \dfrac{\sin B \cos\theta}{\cos B \sin\theta}$

By componendo and dividendo we have

$$\frac{\sinh\phi + \cosh\phi}{\sinh\phi - \cosh\phi} = \frac{\sin B \cos\theta + \cos B \sin\theta}{\sin B \cos\theta - \cos B \sin\theta}$$

or $\dfrac{(e^\phi)}{-(e^{-\phi})} = \dfrac{\sin(B+\theta)}{\sin(B-\theta)}$

or, $\dfrac{e^\phi}{+e^{-\phi}} = \dfrac{\sin(\theta+B)}{+\sin(\theta-B)}$

$\left[\begin{array}{l} e^x = \cosh x + \sinh x \\ e^{-x} = \cosh x - \sinh x \end{array}\right.$

$\sinh\phi - \cosh\phi$
$= -(\cosh\phi - \sinh\phi)$
$= -e^{-\phi}$

or, $e^{2\phi} = \dfrac{\sin(\theta+B)}{\sin(\theta-B)}$

$\log e^{2\phi} = \log\dfrac{\sin(\theta+B)}{\sin(\theta-B)}$

$2\phi \log e = $ ''

$2\phi\,(1) = $ '' proved 2nd part

$\therefore \phi = \frac{1}{2}\log_e \dfrac{\sin(\theta+B)}{\sin(\theta-B)}$

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